BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 11011

SUBJECT: Stability of a Spinning Spacecraft with Flexible Solar Arrays
Case 620

DATE: November 4, 1970

FROM: J. J. Fearnsides
R. J. Ravera
L. E. Voelker

ABSTRACT

An artificial gravity experiment proposed for Skylab B requires vehicle rates on the order of the lowest natural frequency of flexible appendages. The results of a stability analysis of a rotating vehicle with idealized, flexible solar arrays are as follows:

1) The axis of rotation must be the axis of maximum moment of inertia of the undeformed vehicle including the solar arrays.

2) There is a limit on rotation rate above which steady rotation about this axis is not stable.

3) This limit depends on the first mode natural frequency of the solar arrays and on the inertial properties of the vehicle both with and without the solar arrays.

(NASA-CR-111158) STABILITY OF A SPINNING SPACECRAFT WITH FLEXIBLE SOLAR ARRAYS (Bellcomm, Inc.) 10 p

N79-72488

Unclas
00/18 12813

CR-111158 31

[redacted]

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 11011

SUBJECT: Stability of a Spinning Spacecraft with Flexible Solar Arrays
Case 620

DATE: November 4, 1970

FROM: J. J. Fearnsides
R. J. Ravera
L. E. Voelker

MEMORANDUM FOR FILE

Introduction

An artificial gravity experiment proposed for Skylab B requires spin rates on the order of 10 rpm for the centrifugal force to approximate earth gravity. The lowest natural frequency of flexible appendages such as solar arrays is of the same order of magnitude as this rotation rate. An MSC memorandum [1] points out that the dynamic stability of the vehicle as a function of rotation rate depends upon the natural frequency of the appendages and the moments of inertia of the vehicle. In this memorandum, the conditions for stable rotation are determined.

The Model

The vehicle under consideration is Skylab B with symmetric solar arrays. The Skylab without the solar arrays is considered a rigid body. The body-fixed axes x, y, z are aligned with the axes of principal moments of inertia of the composite configuration with origin at the center of mass. The solar arrays are located in the x-y plane with their centers of mass at $\pm$ b on the y-axis. The flexural bending of a solar array is modeled by the displacement of a mass-spring-damper system attached to a massless rigid member, as depicted in Figure 1. The displacements of the two masses are constrained to be antisymmetric in the z direction (for this model), thus applying a moment about the x-axis but not altering the location of the vehicle's center of mass.

The system of equations describing the rotational motion of the spacecraft and the displacements of the masses are non-linear. These equations have an equilibrium solution which is a constant rotation at rate r_o about the z-axis and zero displacement of the masses. Stability is investigated by examining stability of the linearized equations about this equilibrium [2].

Linearized Equations of Motion

The equation of motion for a solar array can be written by summing the forces on the mass m in a rotating coordinate system. These forces include a structural damping term, taken to be a function of the relative velocity of the mass, and a restoring force or stiffness term proportional to the relative displacement. After linearization, this equation becomes

$$\ddot{Z} + \zeta \dot{Z} + \omega_p^2 Z = -b \left(q r_\circ + \dot{p}\right) \tag{1}$$

where

ζ = C/m

ω_p = $(K/m)^{1/2}$, lowest natural frequency of the solar arrays in flexure

$p, q, r_\circ + r$ - components of the angular velocity vector of the vehicle with respect to an inertial reference.

Z - displacement of mass m in the z direction

C - the viscous damping term

K - the stiffness term

Equation (1) governs both masses since they undergo antisymmetric displacements with antisymmetric forces.

The Euler equations governing the rotational motion of the vehicle linearize to

$$I_x \dot{p} + q r_\circ (I_z - I_y) = -2mb \left(\ddot{Z} + r_\circ^2 Z\right) \tag{2a}$$

$$I_y \dot{q} + p r_\circ (I_x - I_z) = 0 \tag{2b}$$

$$I_z \dot{r} = 0 \tag{2c}$$

where I_x, I_y, I_z are the principal moments of inertia of the total vehicle (including the solar array) about the appropriately subscripted principal axes.

Equation (2c) is uncoupled and is satisfied by $r = r_\circ =$ constant. The system of linear equations (1), (2a) and (2b) has the following characteristic equation:

$$\begin{vmatrix} sb & r_\circ b & s^2+\zeta s+\omega_p^2 \\ sI_x & r_\circ(I_z-I_y) & 2mb(s^2+r_\circ^2) \\ r_\circ(I_x-I_z) & sI_y & 0 \end{vmatrix} = 0 \qquad (3a)$$

which, on expansion, can be rewritten in the form

$$0 = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 \qquad (3b)$$

where

$$a_0 = I_y\ (I_x - I_b) \qquad (4a)$$

$$a_1 = \zeta I_x I_y$$

$$a_2 = \omega_p^2 I_x I_y + r_\circ^2[(I_z-I_x)(I_z-I_y)+I_b(I_x-I_y-I_z)] \qquad (4c)$$

$$a_3 = \zeta r_\circ^2(I_z-I_x)(I_z-I_y) \qquad (4d)$$

$$a_4 = r_\circ^2(I_z-I_x)[\omega_p^2(I_z-I_y) - I_b r_\circ^2] \qquad (4e)$$

$I_b = 2mb^2$ is the moment of inertia of the two idealized solar arrays with respect to the center of mass of the composite vehicle.

Stability Criteria

The Routh-Hurwitz criterion gives necessary and sufficient conditions for asymptotic stability. These conditions are [3]:

$$a_0 > 0 \qquad (5a)$$

$$a_1 > 0 \qquad (5b)$$

$$a_4 > 0 \qquad (5c)$$

$$a_1 a_2 - a_0 a_3 > 0 \qquad (5d)$$

$$a_3(a_1 a_2 - a_0 a_3) - a_1^2 a_4 > 0 \qquad (5e)$$

If $a_4 > 0$ and $a_3 > 0$, condition (5d) is implied by condition (5e). Therefore, condition (5d) can be replaced by the condition $a_3 > 0$. Thus conditions (5) may be written as

$$a_0 > 0 \qquad (6a)$$

$$a_1 > 0 \qquad (6b)$$

$$a_3 > 0 \qquad (6c)$$

$$a_4 > 0 \qquad (6d)$$

$$a_3(a_1 a_2 - a_0 a_3) - a_1^2 a_4 > 0 \qquad (6e)$$

which, if satified, automatically imply that $a_2 > 0$ and that condition (5d) is satisfied.

Substituting from equations (4), the conditions (6) become:

$$I_y(I_x - I_b) > 0 \tag{7a}$$

$$\zeta I_x I_y > 0 \tag{7b}$$

$$\zeta r_\circ^2 (I_z - I_x)(I_z - I_y) > 0 \tag{7c}$$

$$r_\circ^2 (I_z - I_x)[\omega_p^2 (I_z - I_y) - I_b r_\circ^2] > 0 \tag{7d}$$

$$I_z(I_z - I_x)(I_z - I_x - I_y)^2 > 0 \tag{7e}$$

For a physically real system, I_x, I_y, I_z, I_b, $r_\circ^2$, ω_p^2, and ζ are all positive, and $I_x - I_b = I_{RBX}$ where I_{RBX} is the moment of inertia of the rigid body about the **x**-axis. Thus, conditions (7a) and (7b) are satisfied. The remaining three conditions can be reduced to the following:

$$I_z > I_x \tag{8a}$$

$$I_z > I_y \tag{8b}$$

$$\omega_p^2 (I_z - I_y) > r_\circ^2 \, I_b \tag{8c}$$

These conditions can also be derived from the work of Likins [4] by the proper substitutions.

Discussion

Conditions (8a), (8b), and (8c) are the conditions for asymptotic stability of rotation about the z-axis.* The first two conditions show that the z-axis must be the axis of maximum moment of inertia, a result obtained by Pringle [5]. Condition (8c) limits the rate of rotation according to the natural frequency of the solar arrays and the moments of inertia of the Skylab and

---

*(that is, no components of rotational perturbation about the x or y axes are sustained.)

the solar arrays. At this critical rate of rotation, $a_4 = 0$ and the system no longer satisfies the necessary conditions for stability, that is, all coefficients of (3b) must be non-zero and of the same sign.

Conditions (8b) and (8c) are identical to two conditions obtained by Lindsay of MSC [1] but (8a) is less restrictive than his corresponding condition. Both Lindsay and Likins have another limit on the rotation rate derived from requiring $a_2 > 0$, but it has been shown that this condition is automatically satisfied if conditions (6) are satisfied.

J. J. Fearnsides

J. J. Fearnsides

R. J. Ravera

R. J. Ravera

L. E. Voelker

L. E. Voelker

JJF
1022-RJR-mef
LEV

Attachments
References
Figure

REFERENCES

1. "Guidance and Control System Feasibility Study of Skylab II System Experiment Module Configuration:, MSC Internal Note No. EG-70-29, July 20, 1970, p. 5.3.1.4.

2. Ogata, K., State Space Analysis of Control System, Prentice-Hall, 1967, pp. 450-452.

3. Ziegler, H., Principles of Structural Stability, Blaisdell, Waltham, Mass., 1968, p. 100.

4. Likins, P. W., "Attitude Stability Criteria for Dual Spin Spacecraft", J. Spacecraft and Rockets, December 1967, pp. 1638-43.

5. Pringle, R., Jr., "On the Stability of a Body with Connected Moving Parts", AIAA J. Vol. 4, No. 8, August 1966, pp. 1395-1404.

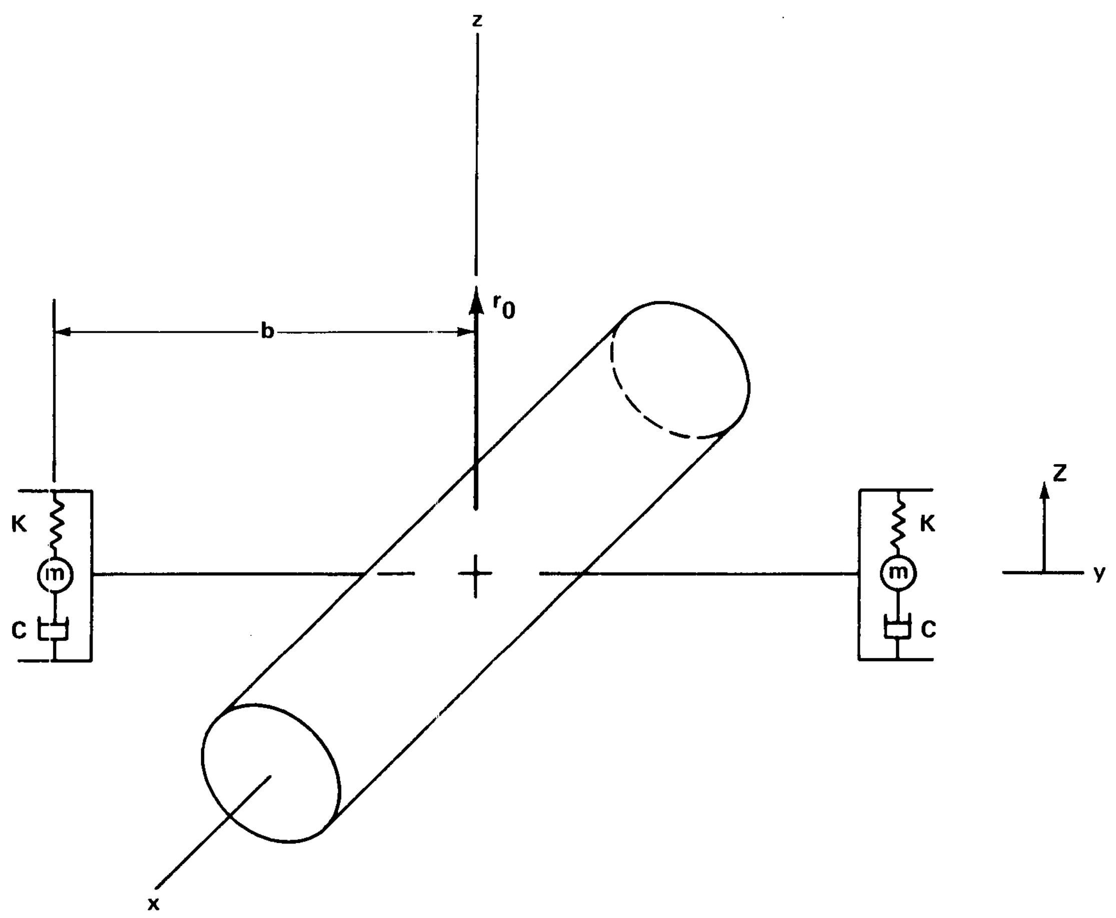

FIGURE 1 - MODEL OF SKYLAB WITH SYMMETRIC FLEXIBLE SOLAR ARRAYS

BELLCOMM, INC.

Subject: Stability of a Spinning Spacecraft with Flexible Solar Arrays
Case 620

From: J. J. Fearnsides
R. J. Ravera
L. E. Voelker

Distribution List

NASA Headquarters

H. Cohen/MLR
J. H. Disher/MLD
W. B. Evans/MLO
J. P. Field, Jr./MLP
T. E. Hanes/MLA
A. S. Lyman/MR
M. Savage/MLT
W. C. Schneider/ML

MSC

K. J. Cox/EG-23
K. L. Lindsay/EG-23
C. F. Lively, Jr./EG-23
A. J. Louviere/EW-6

NAR/Downey

J. A. Jansen/BB-48

Bellcomm

A. P. Boysen
J. P. Downs
D. R. Hagner
W. G. Heffron
J. Z. Menard
I. M. Ross
P. F. Sennewald
J. W. Timko
R. L. Wagner
M. P. Wilson
Departments 2031, 2034 Supervision
Division 101 Supervision
Department 1024 File
Division 102
Central File
Library